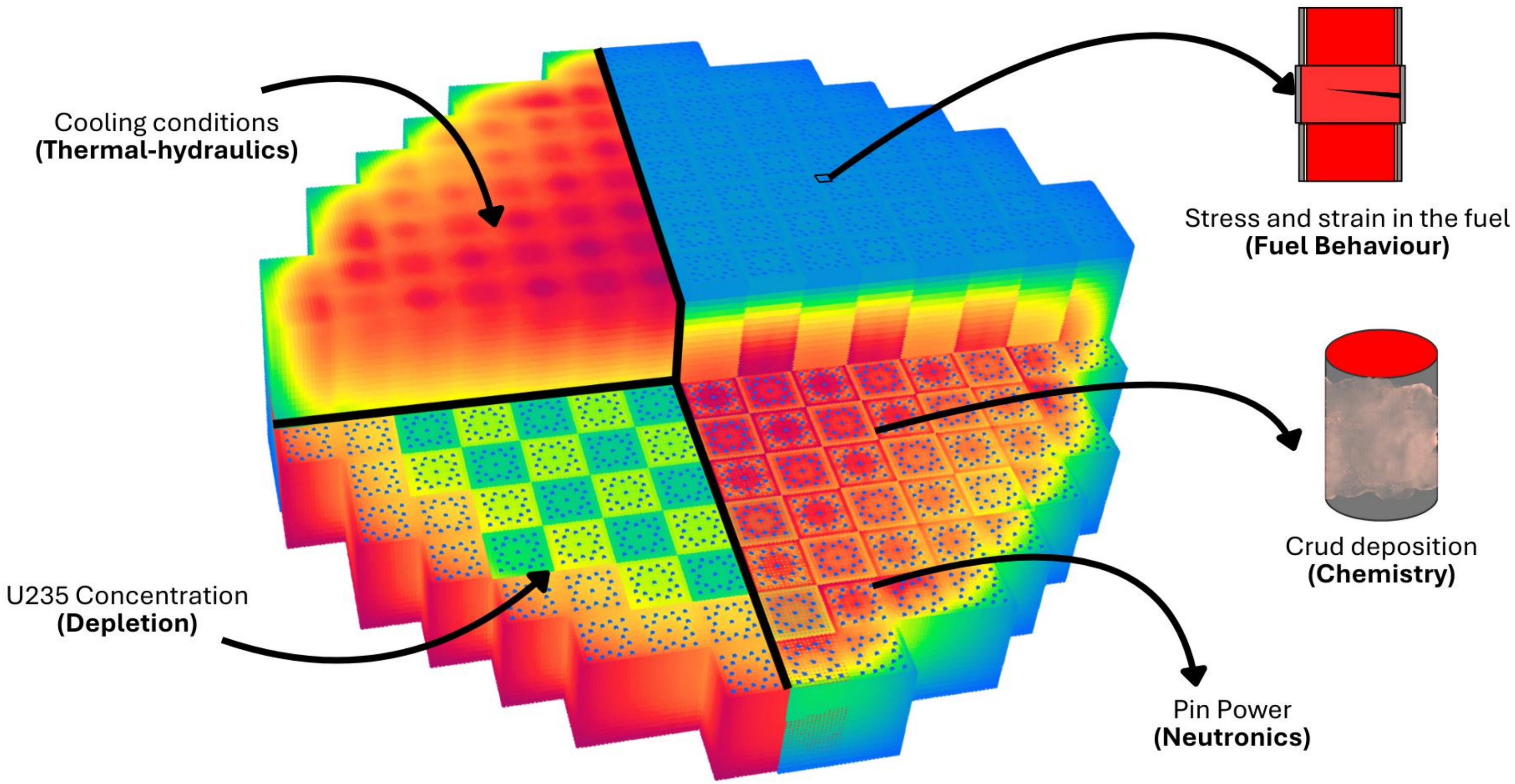

Physics of Nuclear Reactors

Multigroup Theory

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Exercises of today

MULTIGROUP THEORY

10.1 Study of a very large fuel assembly
10.2 Two-group analytical model of a bare cylindrical core

Divide in groups of 5:

- *I will leave you 10 minutes for Exercise 2*
- *We will go through the solution together*
- *I will leave you 10 minutes for Exercise 1*
- *We will go through the solution together*

2. Two-group analytical model of a bare cylindrical core

Exercise description:

Calculate in two-group theory the critical radius of a 3.5m high bare cylindrical core with the cross sections given below:

Group Index	1	2
$\nu\Sigma_f$	0.0085	0.1851
Σ_a	0.0121	0.121
Σ_s (g→g+1)	0.0241	-
χ	1	0
D	1.267	0.354

The cross sections are given in cm^{-1}. We can assumine that both fast and thermal fluxes have the same spatial shape, e.g. the geometrical buckling is energy independent.

Expected results: R=29.6cm; $B^2 = 0.00668$

2. Two-group analytical model of a bare cylindrical core

The first step will be writing the two-group diffusion equations:

$$-D_1\nabla^2 \Phi_1 + \Sigma_{r,1}\Phi_1 = \frac{\chi_1}{k}\left(\nu\Sigma_{f,1}\Phi_1 + \nu\Sigma_{f,2}\Phi_2\right)$$

Leakage **Removal** **Fast fission** **Thermal fission**

$$-D_2\nabla^2 \Phi_2 + \Sigma_{r,2}\Phi_2 = \frac{\chi_2}{k}\left(\nu\Sigma_{f,1}\Phi_1 + \nu\Sigma_{f,2}\Phi_2\right) + \Sigma_{s,1\rightarrow 2}\Phi_1$$

Fission spectrum = 0
In the thermal range

Scattering source

We need then to set our boundary conditions:

Criticality condition with flux going to zero at the boundary

$$\nabla^2 \Phi_i + B^2\Phi_i = 0$$

Geometrical buckling

This allows us to recast the leakage term …

2. Two-group analytical model of a bare cylindrical core

Recasting the leakage term:

$$(\Sigma_{r,1} + D_1B^2)\Phi_1 = \frac{1}{k}(\upsilon\Sigma_{f,1}\Phi_1 + \upsilon\Sigma_{f,2}\Phi_2)$$

$$(\Sigma_{r,2} + D_2B^2)\Phi_2 = \Sigma_{s,1\to 2}\Phi_1$$

Solving for k:

$$k = \frac{\upsilon\Sigma_{f,1}}{\Sigma_{r,1} + D_1B^2} + \frac{\Sigma_{s,1\to 2}}{\Sigma_{r,1} + D_1B^2}\frac{\upsilon\Sigma_{f,2}}{\Sigma_{r,2} + D_2B^2}$$

You are interested in finding the critical size of the system … you set k=1 and solve for B.

$$aB^4 + bB^2 + c = 0 \rightarrow a = D_1D_2$$

$$b = (\Sigma_{r,1} - \upsilon\Sigma_{f,1})D_2 + \Sigma_{r,2}D_1$$

$$c = \Sigma_{r,1}\Sigma_{r,2} - \upsilon\Sigma_{f,1}\Sigma_{r,2} - \Sigma_{s,1\to 2}\upsilon\Sigma_{f,2}$$

$$B^2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = 0.00668$$

For a finite cylinder, the **geometrical buckling** is given by: $B^2 = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2$ So finally, $\boldsymbol{R} = \sqrt{\frac{2.405^2}{B^2 - \left(\frac{\pi}{H}\right)^2}} = \boldsymbol{29.61cm.}$

1. Study of a very large reactor

Exercise description:

Calculate the infinite multiplication constant and the relative group fluxes in a very large fuel assembly with the four-group constants given below:

Group Index	1	2	3	4
$\nu\Sigma_f$	0.0096	0.0012	0.0177	0.1851
Σ_a	0.0049	0.0028	0.0305	0.1210
Σ_s $(g \rightarrow g+1)$	0.0831	0.0585	0.0651	-
χ	0.575	0.425	0	0

The fast flux is normalized to 1.

Use your favorite programming language to solve the linear system of equations

Knowledge to be applied: *u*

Expected results: k_{inf}=1.21500; Φ_1=1; Φ_2=2.4167; Φ_3=1.4788; Φ_4=0.7956;

1. Study of a very large reactor

The first step will be writing the four-group diffusion equations:

- *We neglect leakage because it is a large system*
- *We keep the fission source only in group 1 and 2 (why?)*

$$\Sigma_{r,1}\Phi_1 = \frac{\chi_1}{k}\left(\upsilon\Sigma_{f,1}\Phi_1 + \upsilon\Sigma_{f,2}\Phi_2 + \upsilon\Sigma_{f,3}\Phi_3 + \upsilon\Sigma_{f,4}\Phi_4\right)$$

$$\Sigma_{r,2}\Phi_2 - \Sigma_{s,1\to2}\Phi_1 = \frac{\chi_2}{k}\left(\upsilon\Sigma_{f,1}\Phi_1 + \upsilon\Sigma_{f,2}\Phi_2 + \upsilon\Sigma_{f,3}\Phi_3 + \upsilon\Sigma_{f,4}\Phi_4\right)$$

$$\Sigma_{r,3}\Phi_3 - \Sigma_{s,2\to3}\Phi_2 = 0$$

$$\Sigma_{r,4}\Phi_4 - \Sigma_{s,3\to4}\Phi_3 = 0$$

In matrix form $\mathrm{A}\,\underline{\Phi} = \frac{1}{\mathrm{k}}\,\mathrm{F}\,\underline{\Phi}$ → ***As eigenvalue problem*** $\mathrm{A}^{-1}\mathrm{F}\,\underline{\Phi} = \mathrm{k}\,\underline{\Phi}$

Let us assemble these matrices

1. Study of a very large reactor

Let us assemble the matrix A:

```
A= [sigma_r1,     0,            0,              0; ...
    -sigma_s(1), sigma_r2,      0,              0; ...
    0,           -sigma_s(2),  sigma_r3,        0; ...
    0,           0,            -sigma_s(3),     sigma_r4 ]
```

- Removal term on the diagonal
- Down scattering term on lower diagonal

```
sigma_r1= sigma_a(1)+sigma_s(1);
sigma_r2= sigma_s(2)+sigma_a(2);
sigma_r3= sigma_s(3)+sigma_a(3);
sigma_r4= sigma_s(4)+sigma_a(4);
```

1. Study of a very large reactor

Let us assemble the matrix F:

```
F= [chi(1)*nuf(1), chi(1)*nuf(2), chi(1)*nuf(3), chi(1)*nuf(4); ...
    chi(2)*nuf(1), chi(2)*nuf(2), chi(2)*nuf(3), chi(2)*nuf(4); ...
               0,             0,             0,             0; ...
               0,             0,             0,             0; ...]
```

A matrix

0.088	0	0	0
-0.0831	0.0613	0	0
0	-0.0585	0.0956	0
0	0	-0.0651	0.121

F matrix

0.00552	0.00069	0.010178	0.106433
0.00408	0.00051	0.007523	0.078668
0	0	0	0
0	0	0	0

Using either a direct or iterative solution, the largest eigenvalue and the associated fluxes are:

k_{inf}=1.21500; Φ_1=1; Φ_2=2.4167; Φ_3=1.4788; Φ_4=0.7956

Remember: the exercise tells you to normalize for the fast energy group.